Announcements

• Reading
  • Razavi’s CMOS book chapters 3 & 6
Agenda

- Common-Source Amp Frequency Response
- Open-Circuit Time Constants (OC\(\tau\))
  Bandwidth Estimation Technique
Common-Source Amplifier: Low Frequency Response

\[ v_o = -\frac{g_{m1}}{g_{o1} + g_{o2}} \]

\[ v_i = g_{m1}v_{gs1} \]

\[ v = g_{m1}v_i \]

\[ I_{Bias} \]

\[ R_{in} \]

\[ M1 \]

\[ M2 \]

\[ C_L \]
Common-Source Amplifier: High Frequency Response

Small-Signal Model (Assuming $V_{G2}$ is AC gnd)

KCL @ Node $v_1$: $(v_1 - v_i)G_{in} + v_1sC_{gs1} + (v_1 - v_o)sC_{gd1} = 0$

KCL @ Node $v_o$: $(v_o - v_1)sC_{gd1} + g_m v_1 + v_o(g_o + sC_o) = 0$

where $g_o = g_{o1} + g_{o2}$

After some algebra, we get the exact transfer function:

$$\frac{v_o}{v_i} = \frac{-g_m r_o \left(1 - s \frac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2b}$$

where

$$a = R_{in} \left[C_{gs1} + C_{gd1}\left(1 + g_m r_o\right)\right] + r_o \left(C_{gd1} + C_o\right)$$

and

$$b = R_{in} r_o \left(C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o\right)$$
Common-Source Amp Frequency Response

Exact Transfer Function: \[ \frac{v_o}{v_i} = \frac{-g_m r_o \left(1 - s \frac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b} \]

For the common case when the two poles are real and far apart

Denominator \[ D(s) = \left(1 - \frac{s}{\omega_{p1}}\right)\left(1 - \frac{s}{\omega_{p2}}\right) = 1 - s \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}} \approx 1 - \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}} \]

Thus, \[ \omega_{p1} = -\frac{1}{a} = -\frac{1}{R_{in} \left[C_{gs1} + C_{gd1}(1 + g_{m1} r_o)\right] + r_o \left(C_{gd1} + C_o\right)} \]

and the transfer function can be approximated as a single pole system

\[ A(s) = \frac{v_o}{v_i} \approx \frac{-g_{m1} r_o}{1 + s \left(R_{in} \left[C_{gs1} + C_{gd1}(1 + g_{m1} r_o)\right] + r_o \left(C_{gd1} + C_o\right)\right)} \]
Open-Circuit Time Constants (OCτ)

• Open-circuit time constants technique can be used to estimate bandwidth
  • Much easier than deriving transfer function
  • Accurate for systems with one dominant pole

All - Pole Transfer Function:
\[ \frac{v_o(s)}{v_i(s)} = \frac{a_0}{(\tau_1 s + 1)(\tau_2 s + 1)\ldots(\tau_n s + 1)} \]

Denominator:
\[ b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + 1 \]

Here \( b_n = \prod_{i=1}^{n} \tau_i \) and \( b_1 = \sum_{i=1}^{n} \tau_i \)

A Dominant - Pole System can be approximated as
\[ \frac{v_o(s)}{v_i(s)} \approx \frac{a_0}{b_1 s + 1} = \frac{a_0}{\left(\sum_{i=1}^{n} \tau_i\right)s + 1} \]

Bandwidth \( \omega_h \approx \frac{1}{b_1} = \frac{1}{\sum_{i=1}^{n} \tau_i} = \omega_{h,est} \)
Open-Circuit Time Constants (OCτ)

- To compute time-constants
  1. Compute effective resistance $R_{ko}$ facing each $k$th capacitor with all other caps open-circuited
  2. Form the product $τ_{ko} = R_{ko}C_k$
  3. Sum all $n$ “open-circuit” time constants

\[
ω_{h,est} = \frac{1}{\sum_{k=1}^{n} R_{ko}C_k}
\]
Common-Source Amp w/ OCτ

Small-Signal Model (Assuming VG2 is AC gnd)

• For Cgs

\[
\tau_{lo} = \frac{v_{1o}}{i_{1o}} = \frac{v_{1o}}{1} = R_{in}
\]

\[
R_{1o} = \frac{v_{1o}}{i_{1o}} = \frac{v_{1o}}{1} = R_{in}
\]

\[
\tau_{lo} = R_{in} C_{gs}
\]
Common-Source Amp w/ OCτ

Small-Signal Model (Assuming V_{G2} is AC gnd)

- For $C_{gd1}$

1. $i_{2o} = g_m v_{gs1} + \frac{(v_{2o} + v_{gs1})}{r_o}$
2. $v_{gs1} = -i_{2o} R_{in}$

Plugging (2) into (1) and solving for $\frac{v_{2o}}{i_{2o}}$

\[ R_{2o} = \frac{v_{2o}}{i_{2o}} = R_{in} (1 + g_m r_o) + r_o \]

\[ \tau_{2o} = (R_{in} (1 + g_m r_o) + r_o) C_{gd1} \]
Common-Source Amp w/ OCτ

Small-Signal Model (Assuming VG2 is AC gnd)

- For \( C_0 \)

\[
R_{3o} = \frac{v_{3o}}{i_{3o}} = \frac{v_{3o}}{r_o} = r_o
\]

\[
\tau_{3o} = r_o C_o
\]
Common-Source Amp w/ OCτ

Small-Signal Model (Assuming $V_{G2}$ is AC gnd)

3 Time Constants: $\tau_{1o} = R_{in} C_{gs1}$, $\tau_{2o} = (R_{in} (1 + g_m r_o) + r_o) C_{gd1}$, $\tau_{3o} = r_o C_o$

$$b_1 = \sum_{i=1}^{n} \tau_i = R_{in} C_{gs1} + (R_{in} (1 + g_m r_o) + r_o) C_{gd1} + r_o C_o$$

$$\omega_{h,est} = \frac{1}{b_1} = \frac{1}{R_{in} C_{gs1} + (R_{in} (1 + g_m r_o) + r_o) C_{gd1} + r_o C_o}$$

Exactly the same as what we derived in Slide 6!

$$A(s) = \frac{V_o}{V_i} \approx \frac{-g_m r_o}{1 + s\left[R_{in}(C_{gs1} + C_{gd1}(1 + g_m r_o)) + r_o(C_{gd1} + C_o)\right]}$$
Common-Source Amp w/ Large $R_{in}$

- Example: Using common-source output stage in a 2-stage OpAmp

$$A(s) = \frac{v_o}{v_i} \approx \frac{-g_{m1}r_o}{1 + sR_{in}[C_{gs1} + C_{gd1}(1 + g_{m1}r_o)] + r_o(C_{gd1} + C_o)}$$

with $R_{in} \gg r_o$

$$A(s) = \frac{v_o}{v_i} \approx \frac{-g_{m1}r_o}{1 + sR_{in}[C_{gs1} + C_{gd1}(1 + g_{m1}r_o)]}$$

$$\omega_{p1} = -\frac{1}{R_{in}[C_{gs1} + C_{gd1}(1 + g_{m1}r_o)]}$$

- Dominant pole is formed by input resistance times transistor $C_{gs}$ and $C_{gd}$ which has been multiplied by $1-A_{dc}$
  - $C_{gd}(1-A_{dc})$ is called the Miller capacitance
Common-Source Amp w/ Large $R_{in}$

- What about the second pole?

**Exact Transfer Function:**

$$
\frac{v_o}{v_i} = \frac{-g_m r_o \left( 1 - s \frac{C_{gd1}}{g_{m1}} \right)}{1 + sa + s^2 b}
$$

**Denominator** $D(s) = \left( 1 - \frac{s}{\omega_{p1}} \right) \left( 1 - \frac{s}{\omega_{p2}} \right) = 1 - s \left( \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} \right) + \frac{s^2}{\omega_{p1} \omega_{p2}} \approx 1 - \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1} \omega_{p2}}$

$$
\frac{1}{\omega_{p1} \omega_{p2}} = b = R_{in} r_o \left( C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right)
$$

$$
\omega_{p2} = -\omega_{p1} R_{in} r_o \left( C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right) = -\frac{R_{in} \left[ C_{gs1} + C_{gd1} (1 + g_m r_o) \right]}{R_{in} r_o \left( C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right)}
$$

Assuming that the Miller Cap, $C_{gd1} (1 + g_m r_o)$, dominates

$$
\omega_{p2} \approx -\frac{g_m C_{gd1}}{C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o}
$$
Common-Source Amp w/ Small $R_{in}$

- Example: Source-follower driving the common-source amp

$$A(s) = \frac{v_o}{v_i} \approx \frac{-g_{m1}r_o}{1 + sR_{in}[C_{gs1} + C_{gd1}(1 + g_{m1}r_o)] + r_o(C_{gd1} + C_o)}$$

with $r_o \gg R_{in}$

$$A(s) = \frac{v_o}{v_i} \approx \frac{-g_{m1}r_o}{1 + sr_o(C_{gd1} + C_o)}$$

$$\omega_{p1} = -\frac{1}{r_o(C_{gd1} + C_o)}$$

- Dominant pole is formed by output resistance times output capacitance plus transistor $C_{gd}$
Common-Source Amp w/ Small $R_{in}$

- What about the second pole?

Exact Transfer Function: 
\[
\frac{v_o}{v_i} = \frac{-g_m r_o \left( 1 - s \frac{C_{gd1}}{g_{m1}} \right)}{1 + sa + s^2 b}
\]

Denominator
\[
D(s) = \left( 1 - \frac{s}{\omega_{p1}} \right) \left( 1 - \frac{s}{\omega_{p2}} \right) = 1 - s \left( \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} \right) + \frac{s^2}{\omega_{p1}\omega_{p2}} \approx 1 - \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}
\]

\[
\frac{1}{\omega_{p1}\omega_{p2}} = b = R_{in}r_o \left( C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right)
\]

\[
\omega_{p2} = -\frac{1}{\omega_{p1} R_{in} r_o \left( C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right)} = -\frac{r_o \left( C_{gd1} + C_o \right)}{R_{in} r_o \left( C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right)} = -\frac{r_o \left( C_{gd1} + C_o \right)}{R_{in} \left( C_{gs1} + C_{gd1} \right)}
\]

\[
\omega_{p2} = -\frac{C_{gd1} + C_o}{R_{in} \left( C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o \right)} \approx -\frac{1}{R_{in} \left( C_{gs1} + C_{gd1} \right)} \quad \text{(with large $C_o$)}
\]
Common-Source Amp Frequency Response

\[ A_{dc} = -g_m r_o \]

\[ \omega_z = \frac{g_{m1}}{C_{gd1}} \]

\[ \omega_{p1} = -\frac{1}{R_{in} \left[ C_{gs1} + C_{gd1} \left( 1 + g_{m1} r_o \right) \right]} \]

\[ \omega_{p2} \approx -\frac{g_{m1} C_{gd1}}{C_{gd1} C_{gs1} + C_{gs1} C_o + C_{gd1} C_o} \]

\[ \omega_{p1} \approx -\frac{1}{r_o \left( C_{gd1} + C_o \right)} \]

\[ \omega_{p2} \approx -\frac{1}{R_{in} \left( C_{gs1} + C_{gd1} \right)} \]
Next Time

• Single-Stage Amplifiers (cont.)
  • Common-Drain
  • Common-Gate
  • Cascode Stage

• Differential Pairs