Agenda

• Feedback amplifier examples

• Fully differential circuits

• Common-mode feedback introduction
Non-Inverting Amplifier Example

\[ \frac{v_o}{v_i} = \frac{A(s)}{1 - (-T(s))} \]

If \( T(s) \gg 1 \), then

\[ \frac{v_o}{v_i} \approx \frac{A(s)}{T(s)} = \frac{1}{B(s)} \]

For Error, can write:

\[ \frac{v_o}{v_i} = \frac{1}{B(s)} \left[ \frac{T(s)}{1 + T(s)} \right] = \frac{1}{B(s)} \left[ \frac{1}{1 + \frac{1}{T(s)}} \right] \]

Error \( \propto -\frac{1}{T(s)} \)

Key points:

If you want to amplify your signal: \( B(s) \) must be an attenuator (voltage divider!!)

The error is determined by the overall loop gain: \( T(s) = A(s)B(s) \)

\[ B(s) = \frac{Z_1}{Z_1 + Z_2} = \frac{1}{1 + \frac{Z_2}{Z_1}} \]

\[ \frac{v_o(s)}{v_i(s)} \approx 1 + \frac{Z_2}{Z_1} \]

\( A(s) \) is amplifier response only
Inverting Amplifier: Apply superposition

A(s) and B(s) are sharing some elements!!
A(s) = ? B(s) = ? T(s) = ?

\[ A(s) = \frac{Z_2 A_v}{Z_1 + Z_2} \]

From \( T(s) = A(s)B(s) \)

\[ B(s) = \frac{T(s)}{A(s)} = -\frac{Z_1}{Z_2} \]

\[ \frac{v_o(s)}{v_i(s)} \approx \frac{1}{B(s)} = -\frac{Z_2}{Z_1} \]

\[ \frac{v_0}{v_i} = \frac{A(s)}{1 + T(s)} = -\frac{Z_2 A_v}{1 + \frac{Z_1 A_v}{Z_1 + Z_2}} = -\frac{Z_2}{Z_1} \left( \frac{1}{1 + \frac{Z_2}{Z_1 A_v}} \right) \]
Inverting Amplifier: consider the non-zero output impedance!!

\[ A'(s) = -\frac{(Z_2 + r_0)A_V}{Z_1 + Z_2 + r_0} \]

\[ \frac{v_0'}{v_i} = -\frac{(Z_2 + r_0)A_V}{Z_1 + Z_2 + r_0} \cdot \frac{1}{1 + \frac{Z_1 A_V}{Z_1 + Z_2 + r_0}} \]

\[ T(s) = \frac{Z_1 A_V}{Z_1 + Z_2 + r_0} \]

\[ v_o(s) = \begin{bmatrix} 1 - \frac{r_0}{A_V Z_2} \\ \frac{1}{Z_1} \\ \frac{1}{Z_1 + Z_2 + r_0} \end{bmatrix} \left[ \begin{array}{c} v_o' \\ v_i \\ Z_1 + Z_2 + r_0 \end{array} \right] \]
**Inverting Amplifier:** consider the non-zero output impedance!!

$$\frac{v_o(s)}{v_i} = -\frac{Z_2}{Z_1} \left[ 1 - \frac{r_0}{A_V Z_2} \right] \frac{A_V Z_2}{Z_2 + r_0} \frac{Z_1}{1 + \frac{Z_2 + r_0}{A_V}}$$

The error can be approximated as:

$\text{Error} \approx -\frac{r_0}{A_V Z_2} - \frac{1 + \frac{Z_2 + r_0}{A_V}}{Z_1}$

Determined by the OPAMP open-loop gain

Determined by the OPAMP output impedance
Common-Mode Feedback Techniques for Analog Circuits

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(a) SINGLE-ENDED

(b)

(c) FULLY-DIFFERENTIAL

(d)
Fully-Differential Circuits

In general:
\[ v_{o1} = \frac{v_{o1} - v_{o2}}{2} + \frac{v_{o1} + v_{o2}}{2} = \frac{v_{od}}{2} + v_{oc} \]
\[ v_{o2} = \frac{v_{o2} - v_{o1}}{2} + \frac{v_{o1} + v_{o2}}{2} = -\frac{v_{od}}{2} + v_{oc} \]

Hence

\[
\begin{bmatrix}
  v_{od} \\
  v_{oc}
\end{bmatrix} =
\begin{bmatrix}
  A_{dd} & A_{dc} \\
  A_{cd} & A_{cc}
\end{bmatrix}
\begin{bmatrix}
  v_{id} \\
  v_{ic}
\end{bmatrix}
\]

Differential-mode output

\[
A_{dd} = \frac{v_{od}}{v_{id}} \bigg|_{Vic=0} \quad A_{dc} = \frac{v_{od}}{v_{ic}} \bigg|_{Vid=0}
\]

Common-mode output

\[
A_{cd} = \frac{v_{oc}}{v_{id}} \bigg|_{Vic=0} \quad A_{cc} = \frac{v_{oc}}{v_{ic}} \bigg|_{Vid=0}
\]
Fully-Differential Filters: Effects of current source impedance and mismatches

A very important parameter:

\[ \text{CMRR} = \frac{A_{dd}}{A_{dc}} \]

w/ \( v_{id} = v_{i2} - v_{i1} \) and \( v_{ic} = \frac{v_{i2} + v_{i1}}{2} \)

Solving the circuit:

\[
\begin{align*}
v_{01} &= \frac{g_{m1}g_{m2}Z_1}{g_{m1} + g_{m2} + Y_s} \left[ \left( 1 + \frac{Y_s}{2g_{m2}} \right) v_{id} - \left( \frac{Y_s}{g_{m2}} \right) v_{ic} \right] \\
v_{02} &= \frac{g_{m1}g_{m2}Z_2}{g_{m1} + g_{m2} + Y_s} \left[ -\left( 1 + \frac{Y_s}{2g_{m1}} \right) v_{id} - \left( \frac{Y_s}{g_{m1}} \right) v_{ic} \right]
\end{align*}
\]

\( Y_s \) is the admittance associated with the current source 2IB
Fully-Differential Filters: Non-idealities

Voltage gain: Note the effects of the mismatches, especially in \( A_{dc} \) and \( A_{cd} \)

\[
A_{dd} = \frac{v_{o1} - v_{o2}}{v_{i2} - v_{i1}} \bigg|_{v_{ic} = 0} = \frac{g_{m1} g_{m2}}{g_{m1} + g_{m2} + Y_s} \left[ Z_1 + Z_2 + \frac{Y_s}{2} \left( \frac{Z_1}{g_{m2}} - \frac{Z_2}{g_{m1}} \right) \right]
\]

\[
A_{dc} = \frac{v_{o1} - v_{o2}}{(v_{i2} + v_{i1})/2} \bigg|_{v_{id} = 0} = \frac{g_{m1} g_{m2}}{g_{m1} + g_{m2} + Y_s} \left[ Y_s \left( \frac{Z_2}{g_{m1}} - \frac{Z_1}{g_{m2}} \right) \right]
\]

\[
A_{cd} = \frac{(v_{o2} + v_{o1})/2}{v_{i2} - v_{i1}} \bigg|_{v_{ic} = 0} = \frac{g_{m1} g_{m2}}{g_{m1} + g_{m2} + Y_s} \left( \frac{1}{2} \right) \left[ Z_1 - Z_2 + \frac{Y_s}{2} \left( \frac{Z_1}{g_{m2}} - \frac{Z_2}{g_{m1}} \right) \right]
\]

\[
A_{cc} = \frac{(v_{o2} + v_{o1})/2}{(v_{i2} + v_{i1})/2} \bigg|_{v_{id} = 0} = -\frac{g_{m1} g_{m2}}{g_{m1} + g_{m2} + Y_s} \left( \frac{1}{2} \right) \left[ Y_s \left( \frac{Z_2}{g_{m1}} + \frac{Z_1}{g_{m2}} \right) \right]
\]

\[ CMRR = \frac{A_{dd}}{A_{dc}} \approx \frac{g_{m1} \left( 1 + \frac{Z_1}{Z_2} \right)}{Y_s \left( 1 - \frac{g_{m1} Z_1}{g_{m2} Z_2} \right)} \]
Fully-Differential Circuits

- Ideal voltage gain
  \[ A_{dd} = \frac{v_{01} - v_{02}}{v_{in2} - v_{in1}} = \frac{Z_f}{Z_1} \]

- Ideally even-order distortions are cancelled
- Ideally common-mode signals are rejected
- What sets the output common-mode of these circuits?
  - Function of the amplifier output resistance

Common-mode offsets can impact the performance of the following stages
- Can exceed the common-mode input range of preceding stages
- With finite \( A_{cc} \) can accumulate in a multi-stage amplifier circuit
Fully-Differential Amplifiers: COMMON-MODE DC offset

- If $\Delta I_B$ is positive transistors M3 eventually will be biased in triode region (small resistance)
- dc gain reduces drastically
- Linear range is further minimized
- THD increases
- The common-mode output impedance is the parallel of the equivalent output resistance (M1 and M3) and the parasitic capacitors.
- For large dc gain, the output impedance at nodes v01 and v02 are further increased and $\Delta I_B$ produces a dc offset $= R_{out}\Delta I_B$.
- Large common-mode offsets!
- How this issue can be fixed?
Fully-Differential Amplifiers: Characterization

- Common-mode current offset of 0.01 mA per side is added on purpose.

Tail current is 0.5 mA while the current sources on top are 0.26 mA!

- Differential input voltage is set at 0.
Fully Differential Amplifiers: Characterization

- Offset current is integrated, and output voltage moves upstairs and reach steady state when the current sources on top become equal to 0.25 mA!

- Differential input voltage is set at 0
Fully-Differential Amplifiers: Common-mode Feedback

- Gcmf=100 μA/V.
- If the current offset is 10 μA, then the offset voltage needed for compensation is 100 mV.
- Expected DC outputs after settling are 100 mV.
Fully-Differential Amplifiers: Common-mode Feedback

- $G_{cmf} = 100 \ \mu A/V$.
- If the current offset is 10 \( \mu A \), then the offset voltage needed for compensation is 100 mV.
- Expected DC outputs after settling are 100 mV.
- If necessary to reduce this 100mV offset further, then use a larger $G_{cmf}$.
What is a common-mode feed-back correction circuit?

A common mode feed-back circuit is a circuit sensing the common-mode voltage, comparing it with a proper reference, and feeding back the correcting common-mode signal (both nodes of the fully-differential circuit) with the purpose to cancel the output common-mode current component, and to fix the dc outputs to the desired level.
Fully-Differential Filters: CMFB Principle

- A common-mode feedback loop must be used: Circuit must operate on the common-mode signals only!

- BASIC IDEA: CMFB is a circuit with very small impedance for the common-mode signals but transparent for the differential signals.

- Use a common-mode detector (eliminates the effect of differential signals and detect common-mode signals)

- Analyze the common-mode feedback loop: Large transconductance gain and enough phase margin

- Minimum power consumption

\[ V_{cm} = \frac{V_{o1} + V_{o2}}{2} \]
Next Time

• Common-Mode Feedback Techniques