ECEN474: (Analog) VLSI Circuit Design
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Lecture 18: Feedback & Stability

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Agenda

• Feedback in opamp circuits
• Stability Considerations
  • Nyquist Criteria
  • Phase & Gain Margin
If the OPAMP is not precise, how we can design accurate systems?

Answer is **FEEDBACK!!!**

Examples of our daily life
• Can you shave yourself closing your eyes?
• Can you drive your car closing your eyes?
• Can you adjust the supply voltages without a voltage indicator?
• Can you measure (without any equipment) the magnetic field generated by your cellular phone?
• Can you control properly your daily activities without feedback?

• **If you measure the output at the time you apply the stimuli you can better control the system!!**
Typical values for low-frequency opamps:

$A_v \sim 10^5$

$\omega_p \sim 100$ rad/sec

$r_0 \sim < 100$ Ohms

If $A_v \sim \infty$ then

$v_- = 0$ VIRTUAL GROUND

$v_i/Z_1 = -v_0/Z_2$

$v_o/v_i = -Z_2/Z_1$
If $A_V$ is finite then $v \neq 0$

$$i_1 = -i_2 \text{ if } Z_{in} = \infty$$

or

$$(v_i - (-v_0/A_V))Y_1 = (v_0 - (-v_0/A_V))Y_2$$

$$v_0 = -\frac{Y_1}{Y_2} \left[ \frac{1}{1 + \frac{1}{A_V \left(1 + \frac{Y_1}{Y_2} \right)}} \right] = -\frac{Z_2}{Z_1} \left[ \frac{1}{1 + \frac{1}{A_V \left(1 + \frac{Z_2}{Z_1} \right)}} \right]$$

Using the approximation $\frac{1}{1 + x} \approx 1 - x$ for small $x$

$$\text{Error} = -\frac{1}{A_V \left(1 + \frac{Z_2}{Z_1} \right)} = -\frac{1 + \frac{s}{\omega_p}}{A_{DC} \left(1 + \frac{Z_2}{Z_1} \right)}$$

$$A_V = \frac{A_{VDC}}{1 + \frac{s}{\omega_p}}$$

$R_{out} = 0$ and $Z_{in} = \infty$
FEEDBACK:

• If you measure the output at the time you apply the stimuli you can better control the system!!

Definitions:

A(s): Amplifier gain (very large but not very well controlled)
B(s): Feedback Factor (Very well controlled)
T(s)=A(s)B(s): Loop Gain (Extremely important parameter!!)

Applying Mason Rule:

\[
\frac{v_o}{v_i} = \frac{Direct\ trajectory}{1-loop\ gain} = \frac{A(s)}{1-(-T(s))} = \frac{A(s)}{1+A(s)B(s)}
\]
Feedback Configuration

Here $f = \text{feedback factor (B(s) in previous slides)}$

\[
a(s) = \frac{V_o}{V_e}(s) = \frac{a_o}{1 - \frac{s}{p_1}}
\]

\[
A_{CL}(s) = \frac{V_o}{V_i}(s) = \frac{a(s)}{1 + a(s)f} = \frac{a_o}{1 + a_of} \frac{1}{1 - \frac{s}{(1 + a_of)p_1}}
\]
Gain-Bandwidth

\[ 20 \log |a(j \omega)| \]

\[ 20 \log \frac{a_o}{1 + a_o f} \]

\[ 20 \log |A_{cl}(j \omega)| \]

\[ \omega_t \]

Gain magnitude, dB

log scale

[Karsilayan]
Instability and the Nyquist Criterion

Transfer function of a 3-pole amplifier:

\[ a(s) = \frac{a_0}{\left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)\left(1 - \frac{s}{p_3}\right)} \]

Nyquist criterion for stability of the amplifier:

Consider a feedback amplifier with a stable \( T(s) \). If the Nyquist plot of \( T(j\omega) \) encircles the point \((-1,0)\), the feedback amplifier is unstable.

Recall \( T(s) \) is the loop gain

\[ T(s) = A(s)B(s) = a(s)f \]
Magnitude & Phase

3-pole amplifier

$|a(j\omega)|$, dB

20 log $a_0$

20 log $a_{180}$

-20 dB/dec

-40 dB/dec

-60 dB/dec

$\omega$

log scale

$\omega$

$\angle a(j\omega)$

[Karsilayan]
Magnitude & Phase

\[ T(s) = a(s)f_1 \]

\[ |T(j\omega)|, \text{ dB} \]

20 log \( a_0 f_1 \)

-20 dB/dec

20 log \( a_{180} f_1 \)

-40 dB/dec

\[ \omega \]

\[ |p_1| \quad |p_2| \quad \omega_{180} \quad |p_3| \quad -60 \text{ dB/dec} \]

\[ \angle T(j\omega) \]

[log scale]
Nyquist Plot

Frequency Sweep of Loop Gain, $T(s)$

$T(s) = a(s)f_1$

[Karsilayan]
Magnitude & Phase

\[ T(s) = a(s)f_2 \]

[Karsilayan]
Nyquist Plot

\[ T(s) = a(s)f_2 \]

[Karsilayan]
Gain & Phase Margin

$|a(j\omega)|, \text{ dB}$

$|T(j\omega)|=0 \text{ dB}$

$20 \log a_o$

$20 \log a_{180}$

$\omega$ axis for $|T(j\omega)|$

Gain margin

$20 \log \frac{1}{f}$

$\omega$

$log scale$

$\angle a(j\omega), \angle T(j\omega)$

Phase margin

[Karsilayan]
**Stability Criteria**

Nyquist:

\[ |T(j\omega_{180})| = a_{180}f < 1 \quad \Rightarrow \quad \text{Stable} \]  

[Karsilayan]

Gain Margin (**GM**):

\[
GM = 20 \log \frac{1}{|T(j\omega_{180})|} = -20 \log |T(j\omega_{180})|
\]

\[ GM > 0 \quad \Rightarrow \quad \text{Stable} \]

Phase Margin (**PM**):

\[
PM = 180^\circ + \angle T(j\omega_0)
\]

\[ PM > 0 \quad \Rightarrow \quad \text{Stable} \]
Phase Margin

\[ |T(j\omega_0)| = 1 \Rightarrow |a(j\omega_0)|f = 1 \Rightarrow |a(j\omega_0)| = \frac{1}{f} \]

PM = 45°  \Rightarrow  \angle T(j\omega_0) = -135° ,  \quad A_{cl}(j\omega_0) = \frac{a(j\omega_0)}{1 + T(j\omega_0)}

\[ A_{cl}(j\omega_0) = \frac{a(j\omega_0)}{1 + e^{-j135^\circ}} = \frac{a(j\omega_0)}{1 - 0.7 - 0.7j} \]

\[ |A_{cl}(j\omega_0)| = \frac{|a(j\omega_0)|}{|0.3 - 0.7j|} = \frac{1}{0.76f} = 1.3 \]

PM = 30°  \Rightarrow  \angle T(j\omega_0) = -150° ,  \quad |A_{cl}(j\omega_0)| = 1.92/f

PM = 60°  \Rightarrow  \angle T(j\omega_0) = -120° ,  \quad |A_{cl}(j\omega_0)| = 1/f

PM = 90°  \Rightarrow  \angle T(j\omega_0) = -90° ,  \quad |A_{cl}(j\omega_0)| = 0.7/f
Closed-Loop Step Response

PM = 30°

PM = 45°

PM = 60°

PM = 90°
Non-Inverting Amplifier Example

\[
\frac{v_o}{v_i} = \frac{A(s)}{1 - (-T(s))}
\]

If \( T(s) \gg 1 \), then
\[
\frac{v_o}{v_i} \approx \frac{A(s)}{T(s)} = \frac{1}{B(s)}
\]

For Error, can write:
\[
\frac{v_o}{v_i} = \frac{1}{B(s)} \left[ \frac{T(s)}{1 + T(s)} \right] = \frac{1}{B(s)} \left[ \frac{1}{1 + \frac{1}{T(s)}} \right]
\]

\( \text{Error} \propto - \frac{1}{T(s)} \)

Key points:

If you want to amplify your signal: \( B(s) \) must be an attenuator (voltage divider!!)

The error is determined by the overall loop gain: \( T(s) = A(s)B(s) \)

\[
B(s) = \frac{Z_1}{Z_1 + Z_2} = \frac{1}{1 + \frac{Z_2}{Z_1}}
\]

\[
\frac{v_o(s)}{v_i(s)} \approx 1 + \frac{Z_2}{Z_1}
\]

\( A(s) \) is amplifier response only

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**Inverting Amplifier: Apply superposition**

**A(s) and B(s) are sharing some elements!!**

\[ A(s) = \frac{Z_2 A_V}{Z_1 + Z_2} \]

**From** \( T(s) = A(s)B(s) \)

\[ B(s) = \frac{T(s)}{A(s)} = -\frac{Z_1}{Z_2} \]

\[ \frac{v_o(s)}{v_i(s)} \approx \frac{1}{B(s)} = -\frac{Z_2}{Z_1} \]

\[ v_0 = \frac{A(s)}{1 + T(s)} = -\frac{Z_2 A_V}{1 + \frac{Z_1 A_V}{Z_1 + Z_2}} = -\frac{Z_2}{Z_1} \left( 1 + \frac{Z_2}{1 + \frac{Z_1}{A_V}} \right) \]
Next Time

• Common-Mode Feedback Techniques