Agenda

• MOSFET Noise
• Filtered Noise
• OTA Noise Example
Resistor Noise Model

- An equivalent voltage or current generator can model the resistor thermal noise

\[
V_{Rn}^2 = P_n R = 4kTR\Delta f
\]

\[
I_{Rn}^2 = \frac{P_n}{R} = \frac{4kT}{R} \Delta f
\]

- Recall the PSD is white (uniform w/ frequency)
Diode Noise Model

- Shot noise in diodes is caused by pulses of current from individual carriers in semiconductor junctions.

- White spectral density

\[ r_d = \frac{kT}{qI_D} \quad \text{(noiseless)} \]

\[ V_d^2(f) = 2kTr_d \]

\[ r_d = \frac{kT}{qI_D} \quad \text{(noiseless)} \]

\[ I_d^2(f) = 2qI_D \]

- Where \( q = 1.6 \times 10^{-19} \text{C} \) and \( I_D \) is the diode DC current.
Thermal Noise

\[ i_d^2 = \frac{4kT}{R_{DS}} \]

\[ R_{DS} \approx \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_T - V_{DS})} \]
White Noise

@ Triode region

\[ i_d^2 = \left[ 4kT \mu C_{ox} \left( \frac{W}{L} \right) \right] \left( V_{GS} - V_T - V_{DS} \right) \]

Low current noise => W/L \( \downarrow \) => \( g_m \) or \( g_o \) \( \downarrow \)

@ Saturation

\[ g_o = \frac{1}{R_{DS}} \rightarrow \frac{2}{3} g_m \]

\[ i_d^2 = \frac{8}{3} kT g_m \]

\[ \Rightarrow i_d^2 = \left( \frac{8kT}{3} \right) \left( \mu C_{ox} \right) \left( \frac{W}{L} \right) \left( V_{GS} - V_T \right) \]
MOSFET 1/f (Flicker) Noise

• Caused by traps near Si/SiO₂ interface that randomly capture and release carriers

\[ i_d^2(f) = \frac{K_F g_m^2}{WLC_{ox} f} \]

• \( K_F \) is strongly dependent on the technology
1/f Noise Corner Frequency

• This is the frequency at which the flicker noise density equals the thermal noise density

\[
\frac{K_F g_m^2}{WLC_{ox} f_{co}} = 4kT \gamma g_m
\]

\[
f_{co} = \frac{K_F}{4kT \gamma C_{ox}} \frac{g_m}{WL} = \frac{K_F}{4kT \gamma C_{ox}} \frac{1}{L} \left( \frac{g_m}{I_D} \right) \left( \frac{I_D}{W} \right)
\]

• For a given \( g_m/I_D \) (which sets \( I_D/W \)), the only way to reduce \( f_{co} \) is to use longer channel devices
Output and input referred noise

Current noise is the real one

Thermal Noise
\[ i_d = g_m V_{gs} \]
\[ i_d = g_m V_{gs}^2 \]

\[ v_{eq}^2 = \frac{K_F g_m^2}{WLC_{ox} f} \]

Voltage noise representation is an artifact to facilitate system analysis

\[ v_{eq}^2 = \frac{K_F g_m^2}{WLC_{ox} f} \left( \frac{1}{g_m^2} \right) \]

Flicker Noise
\[ i_d = \frac{K_F g_m^2}{WLC_{ox} f} \]

Reflected to the input
\[ v_{eq}^2 = \frac{K_F}{C_{ox}} \left( \frac{1}{f} \right) \]
Equivalent input referred voltage noise

\[ V_{eq}^2 = \frac{i_{dth}^2 + i_{df}^2}{g_m^2} \]

Equivalent input referred noise voltage means that all current noise sources are accounted as drain current and represented by an “equivalent” noise voltage at transistor gate.

\[ V_{eq}^2 = \frac{8 kT}{3 g_m} + \frac{K_F}{C_{ox} \cdot WL \cdot f} \]

\[ V_{eq_{total}} (\text{RMS}) = \sqrt{\int_{BW}^{\text{f}} V_{eq(f)}^2 df} \]
NOISE COMPONENTS (values provided are for a 0.8 \(\mu\)m technology)

![Diagram of noise components]

Noise density (V\(^2\)/Hz)

\[
v_{eq}^2 = v_{th}^2 + v_{1/f}^2
\]

\[
v_{eq}^2 = \frac{8}{3} \frac{kT}{g_m} df + \frac{K_F}{WLC_{OX}f} df
\]

\[
\frac{K_F}{C_{OX}} = 9.8 \times 10^{-9} V^2 / \mu m - Hz (NMOS)
\]

\[
= 0.5 \times 10^{-9} V^2 / \mu m - Hz (PMOS)
\]

FOR LOW-FREQUENCY APPLICATIONS,
WHEREIN 1/F NOISE IS DOMINANT,
PMOS DEVICES MUST BE USED.
Filtered Noise

\[ v_{ni}^2(f) \xrightarrow{A(s)} v_{no}^2(f) = |A(j2\pi f)|^2 v_{ni}^2(f) \]
\[ v_{no}^2(f) = |A(j2\pi f)| v_{ni}(f) \]

- Noise output spectral density is a function only of the magnitude of the transfer function, and not its phase.
- With multiple uncorrelated noise sources, combined output is also uncorrelated.

\[ v_{n1}^2(f) \xrightarrow{A_1(s)} v_{n2}^2(f) \xrightarrow{A_2(s)} v_{n3}^2(f) \xrightarrow{A_3(s)} v_{no}^2(f) = \sum_{i=1,2,3} |A_i(j2\pi f)|^2 v_{ni}^2(f) \]
First-Order RC Circuit Example

What is the total output noise power?
First-Order RC Circuit Example

\[ A(s) = \frac{v_{out}(s)}{v_R(s)} = \frac{1}{1 + sRC} \]

\[ v_{out}^2(f) = |A(j2\pi f)|^2 v_R^2(f) = \frac{1}{1 + 4\pi^2 f^2 R^2 C^2} 4kTR \]

To calculate Total Noise Power integrate over all frequencies

\[ v_{out}^2 = \int_{0}^{\infty} \frac{4kTR}{1 + 4\pi^2 f^2 R^2 C^2} \]

Using \[ \int \frac{dx}{x^2 + 1} = \tan^{-1} x \]

\[ v_{out}^2 = \frac{2kT}{\pi C} \tan^{-1} (2\pi fRC) \bigg|_{f=0}^{f=\infty} = \frac{2kT}{\pi C} \left( \frac{\pi}{2} - 0 \right) = \frac{kT}{C} \]
Noise is generated by $R$ but integrated noise is function of $C$ (??)

\[ v_{total}^2 = \int_{0}^{\infty} \left( \frac{1}{1 + (\omega RC)^2} \right) (4kTR) \, df = \frac{kT}{C} \]

To get more insight, let's have a closer look on the operations!

Notice that:
When $R$ increases thermal noise increases too but the corner frequency decreases, leading to a constant area under the curves!
Noise Bandwidth

- The noise bandwidth is equal to the frequency span of a brickwall filter having the same output noise rms value

\[ v_0^2 B_n = \int_0^\infty v_{no}^2 \, df \]

For a first-order filter \( B_n = \frac{\pi}{2} \frac{\omega_p}{\omega_p} \)

Validating with previous slides derivation:

Total Noise Output = \( v_0^2 B_n = (4kTR) \left( \frac{\pi}{2} \right) \left( \frac{1}{2\pi RC} \right) = \frac{kT}{C} \)
Output referred noise: Take advantage of SYMMETRIES!

Output referred current noise density

Superposition: Every transistor contributes; consider one at the time.

Analysis: You can use standard circuit analysis techniques but at the end of the day you have to consider POWER.

Output noise density: Each noise component represent the RMS value of random uncorrelated noise! Then add the power noise components

\[ i_{\text{out}1}^2 = \frac{8}{3} kT g_{m1} \]
Output referred noise: Take advantage of SYMMETRIES!

Output referred current noise density due to the P-type devices:

Left hand side transistor:

$$i_{out2}^2 \approx i_{d2}^2 = \frac{8}{3} kTg_{m2}$$

Right hand side transistor

$$i_{out2}^2 = \frac{8}{3} kTg_{m2}$$
Output and input referred noise

Output referred current noise density

\[ i_{\text{out}}^2 = 2 \left( \frac{8}{3} kT g_{m1} \right) + 2 \left( \frac{8}{3} kT g_{m2} \right) \]

Input referred noise density (V²/Hz)

\[ v_{\text{in,eq}}^2 = 2 \left( \frac{8}{3} \frac{kT}{g_{m1}} \right) + 2 \left( \frac{8}{3} \frac{kT}{g_{m1}} \frac{g_{m2}}{g_{m1}} \right) \]

In this case, noise due to the current source is mainly common-mode noise

Be careful because this is not always the case!
Integrated Input referred noise

\[ \text{Input referred thermal noise density (V}^2/\text{Hz}) \]
\[ v_{\text{in,eq}}^2 = 2 \left( \frac{8}{3} \frac{kT}{g_{m1}} \right) + 2 \left( \frac{8}{3} \frac{kT}{g_{m1} g_{m2}} \right) \]

\[ \text{Input referred noise level (volts)} \]
\[ \text{Noise}(V_{\text{RMS}}) = \sqrt{\int_{BW} V_{\text{in,eq}}^2 \, df} \]

Example: for thermal noise, the noise level becomes (assuming a single-pole system)
\[ \text{Noise}(V_{\text{RMS}}) \approx \sqrt{\frac{16kT}{3}} \sqrt{\frac{1}{g_{m1}}} \sqrt{1 + \frac{g_{m2}}{g_{m1}}} \left( \sqrt{\frac{\pi}{2}} \frac{BW}{BW} \right) \]

\[ \text{Noise}(V_{\text{RMS}}) \approx \sqrt{\frac{8kT}{g_{m1}}} \sqrt{1 + \frac{g_{m2}}{g_{m1}}} \left( \sqrt{BW} \right) \]

4kT \approx 16 \times 10^{-21} \text{ coul.V}

I should advise you to use:
Next Time

• OTAs