Announcements

- Reading
  - Razavis’ CMOS Book Chapter 7
Agenda

- Noise Types
- Noise Properties
- Resistor Noise Model
- Diode Noise Model
Noise Significance

• Why is noise important?
  • Sets minimum signal level for a given performance parameter
  • Directly trades with power dissipation and bandwidth
• Reduced supply voltages in modern technologies degrades noise performance

\[ \text{Signal Power } \propto (\alpha V_{dd})^2 \Rightarrow \text{SNR} = \frac{P_{\text{sig}}}{P_{\text{noise}}} \propto \left( \frac{\alpha V_{dd}}{V_{\text{noise}}} \right)^2 \]

• Noise is often proportional to kT/C
  • Increasing capacitance to improve noise performance has a cost in increase power consumption for a given bandwidth
Interference Noise

- Interference “Man-Made” Noise
  - Deterministic signal, i.e. not truly “random”
    - Could potentially be modeled and predicted, but practically this may be hard to do
  - Examples
    - Power supply noise
    - Electromagnetic interference (EMI)
    - Substrate coupling
  - Solutions
    - Fully differential circuits
    - Layout techniques

- Not the focus of this lecture
  - Unless the deterministic noise is approximated as a random process
Inherent Noise

• “Electronic” or “Device” Noise
  • Random signal
  • Fundamental property of the circuits
  • Examples
    • Thermal noise caused by thermally-excited random motion of carriers
    • Flicker (1/f) noise caused by material defects
    • Shot noise caused by pulses of current from individual carriers in semiconductor junctions
  • Solutions
    • Proper circuit topology
    • More power!!!

• Is the focus of this lecture
Noise Properties

- **Noise is random**
  - Instantaneous noise value is unpredictable and the noise must be treated statistically
  - Can only predict the average noise power
  - Model with a Gaussian amplitude distribution
  - Important properties: mean (average), variance, power spectral density (noise frequency spectrum)
RMS Value

• If we assume that the noise has zero mean (generally valid)
• RMS or “sigma” value is the square-root of the noise variance over a suitable averaging time interval, \( T \)

\[
V_{n(rms)} = \left( \frac{1}{T} \int_0^T v_n^2(t) dt \right)^{1/2}
\]

• Indicates the normalized noise power, i.e. if \( v_n(t) \) is applied to a \( 1\Omega \) resistor the average power would be

\[
P_n = \frac{V_{n(rms)}^2}{1\Omega} = V_{n(rms)}^2
\]
Signal-to-Noise Ratio (SNR)

\[ SNR \equiv 10 \log \left( \frac{\text{signal power}}{\text{noise power}} \right) \]

For a signal with normalized power of \( V_{x(rms)}^2 \)

\[ SNR \equiv 10 \log \left[ \frac{V_{x(rms)}^2}{V_{n(rms)}^2} \right] = 20 \log \left[ \frac{V_{x(rms)}}{V_{n(rms)}} \right] \]

- Quantified in units of dB
Thermal Noise Spectrum

- The power spectral density (PSD) quantifies how much power a signal carries at a given frequency
- Thermal noise has a uniform or “white” PSD

The total average noise power $P_n$ in a particular frequency band is found by integrating the PSD

$$P_n = \int_{f_1}^{f_2} PSD(f)df$$

For white noise spectrum: $P_n = n_0(f_2 - f_1) = n_0 \Delta f$
Thermal Noise of a Resistor

• The noise PSD of a resistor is

\[ PSD(f) = n_0 = 4kT \]

where \( k \) is the Boltzmann constant and \( T \) is the absolute temperature (K)

• The total average power of a resistor in a given frequency band is

\[ P_n = \int_{f_1}^{f_2} 4kTdf = 4kT(f_2 - f_1) = 4kT\Delta f \]

• Example: \( \Delta f = 1\) Hz → \( P_n = 4 \times 10^{-21} \) W = -174 dBm
Resistor Noise Model

- An equivalent voltage or current generator can model the resistor thermal noise

\[ V_{Rn}^2 = P_n R = 4kTR\Delta f \]
\[ I_{Rn}^2 = \frac{P_n}{R} = \frac{4kT}{R} \Delta f \]

- Recall the PSD is white (uniform w/ frequency)
Noise Summation

\[ v_{n1}(t) \star v_{n2}(t) \star \rightarrow ^{+} v_{no}(t) \]

**Voltage**

\[ v_{no}(t) = v_{n1}(t) + v_{n2}(t) \]

\[ V_{no(rms)}^2 = \frac{1}{T} \int_{0}^{T} [v_{n1}(t) + v_{n2}(t)]^2 dt \]

\[ V_{n1(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2 + \frac{2}{T} \int_{0}^{T} v_{n1}(t)v_{n2}(t) dt \]

- Same procedure applies to noise current summing at a node
Correlation

• Last term describes the correlation between the two signals, defined by the correlation coefficient, $C$

$$C = \frac{1}{T} \int_{0}^{T} v_{n1}(t)v_{n2}(t) dt$$

$$V_{no(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2 + 2CV_{n1(rms)}V_{n2(rms)}$$

• Correlation always satisfies $-1 \leq C \leq 1$
  • $C=+1$, fully-correlated in-phase (0°)
  • $C=-1$, fully-correlated out-of-phase (180°)
  • $C=0$, uncorrelated (90°)
Uncorrelated Signals

• For two uncorrelated signals, the mean-squared sum is given by

\[ V_{no(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2 \]

Add as though they were vectors at right angles

• For two fully correlated signals, the mean-squared sum is given by

\[ V_{no(rms)}^2 = \left( V_{n1(rms)} \pm V_{n2(rms)} \right)^2 \]

Sign is determined by phase relationship
RMS values add linearly (aligned vectors)
Noise Example #1: Two Series Resistors

\[ v_{n1(rms)}^2 = v_{n1(rms)}^2 + v_{n2(rms)}^2 + 2Cv_{n1(rms)}v_{n2(rms)} \]

- The noise of the two resistors is uncorrelated or statistically independent, so \( C=0 \)

\[ v_{n(rms)}^2 = v_{n1(rms)}^2 + v_{n2(rms)}^2 = 4kT(R_1 + R_2)\Delta f \]

- Always add independent noise sources using mean squared values
  - Never add RMS values of independent sources
Noise Example #2: Voltage Divider

- Lets compute the output voltage: **Apply superposition** (noise sources are small signals, you can use small signal models)!

\[
V_0 = \left( \frac{R_2}{R_1 + R_2} \right)V_{\text{in}} + \left( \frac{R_2}{R_1 + R_2} \right)V_{n1} + \left( \frac{R_1}{R_1 + R_2} \right)V_{n2}
\]

Above is what you do for deterministic signals, but we cannot do this for the resistor noise

But noise is a random variable, power noise density has to be used rather than voltage; then the output referred noise density (noise in a bandwidth of 1 Hz) becomes

\[
V_{0n}^2 = \left( \frac{R_2}{R_1 + R_2} \right)^2 V_{n1}^2 + \left( \frac{R_1}{R_1 + R_2} \right)^2 V_{n2}^2
\]

\[
V_{0n}^2 = \left( \frac{R_2}{R_1 + R_2} \right)^2 4kTR_1 + \left( \frac{R_1}{R_1 + R_2} \right)^2 4kTR_2
\]

**General Case:**

\[
V_{on,f}^2(f) = \sum_{x} \left| H_x(s) \right|^2 v_x^2(f)
\]
Diode Noise Model

- Shot noise in diodes is caused by pulses of current from individual carriers in semiconductor junctions.
- White spectral density

\[
V^2_d(f) = 2kT r_d
\]

\[
I^2_d(f) = 2qI_D
\]

- Where \( q = 1.6 \times 10^{-19} \text{C} \) and \( I_D \) is the diode DC current.

[Johns]
Next Time

- Noise in MOSFETs
- Noise Analysis