Equalization
Introduction

- When the channel is not ideal, or when signaling is not Nyquist, there is ISI at the receiver side.
- The optimum receiver is a ML Sequence Estimator, which often is too complex.
- In practice, we *equalize* the channel response using an equalizer.

\[ |P(f)| = |H(f)| = \sqrt{G_{rc}(f)}, \]  
where \( G_{rc}(f) \) is the raised-cosine response.
The Zero-Forcing Equalizer

The overall response at the detector input must satisfy Nyquist’s criterion for no ISI:

\[ G_{rc}(f) = |P(f)| \cdot |C(f)| \cdot |H(f)| \cdot |E(f)| \]
\[ = G_{rc}(f) \cdot |C(f)| \cdot |E(f)| \Rightarrow \]

\[ E(f) = \frac{1}{|C(f)|} \cdot e^{-ja(f)} \text{ where } a(f) \text{ is the phase of } C(f) \]

The noise variance at the output of the equalizer is:

\[ \sigma_e^2 = \frac{N_0}{2} \cdot \int_{-\infty}^{\infty} \frac{G_{rc}(f)}{|C(f)|^2} df \]

If the channel has spectral nulls, there may be significant noise enhancement.
Transversal-Filter Zero-Forcing Equalizer

If $T_s < T$, we have a fractionally-spaced equalizer

\[ e(t) = \sum_{n=-N}^{N} c_n \delta(t - nT_s) \iff E(f) = \sum_{n=-N}^{N} c_n e^{-j2\pi fnT_s} \]

\[ g(kT) = \sum_{n=-N}^{N} c_n x[(kT - nT_s)], \quad k = 0, \pm 1, \pm 2, \ldots, \pm N \]

For no ISI, let:

\[ g(kT) = \begin{cases} 
1 & k = 0 \\
0 & k = \pm 1, \pm 2, \ldots, \pm N 
\end{cases} \]
Thus...

\[ X \cdot c = g \implies c = X^{-1} \cdot g \]

Example: Consider a baud-rate sampled equalizer for a system for which

\[ x(t) = \frac{1}{1 + \left( \frac{t}{T} \right)^2} \]

Design a zero-forcing equalizer having 5 taps.

\[
X = \begin{bmatrix}
1.0 & 0.5 & 0.2 & 0.1 & 0.059 \\
0.5 & 1.0 & 0.5 & 0.2 & 0.1 \\
0.2 & 0.5 & 1.0 & 0.5 & 0.2 \\
0.1 & 0.2 & 0.5 & 1.0 & 0.5 \\
0.059 & 0.1 & 0.2 & 0.5 & 1.0
\end{bmatrix}, \quad
\begin{bmatrix}
g_0 \\
g_1 \\
g_2 \\
g_3 \\
g_4
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
1 \\
0 \\
0
\end{bmatrix}
\]

\[
c = X^{-1} \cdot g = \begin{bmatrix}
0.1142 \\
-0.7796 \\
1.7339 \\
-0.7796 \\
0.1142
\end{bmatrix}
\]
To Probe Further...

- Zero-forcing equalizers ignore the additive noise and may significantly amplify noise for channels with spectral nulls.
- Minimum-mean-square error (MMSE) equalizers minimize the mean-square error between the output of the equalizer and the transmitted symbol. They require knowledge of some auto and cross-correlation functions, which in practice can be estimated by transmitting a known signal over the channel.
- *Adaptive equalizers* are needed for channels that are time-varying.
- *Blind equalizers* are needed when no preamble is allowed.
- *Decision-feedback equalizers* (DFE’s) use tentative symbol decisions to eliminate ISI.
- Ultimately, the optimum equalizer is a *maximum-likelihood sequence estimator*.